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MATHEMATICAL MODEL OF THE LORENZ CURVE: ON BALANCING THE WEALTH OF COMMUNITIES

MIGDAT HODŽIĆ

Dedicated to the 75th birthday of our dear Professor Mirjana Vuković

ABSTRACT. This paper presents a new (i) mathematical approach to modeling the wealth of a community and (ii) a method of balancing this wealth, with the idea of fair distribution among community members. The paper evaluates how far the wealth of a community (country, the world) is from the ideal, measured by the so-called Lorenz curve, known from the domain of economics. The work can also be interpreted as a detailed mathematical model of the Lorenz curve with a recursive algorithm for "correcting" that curve, which is ideally a line in a two-dimensional space (number of community members, their wealth). In economic literature, it is easy to find descriptions of Lorenz curves for various countries, in the form of a set of straight lines (which approximate an otherwise non-linear function) for various groups of wealth. Another method based on the Lorenz curve is the so-called The Gini index, which also measures differences in wealth. Unlike such standard models, our work presents a detailed mathematical model of Lorenc curve as well as wealth balancing. Mathematically, the algorithm describes in detail a recursive method that "corrects" the central part of the Lorenz curve, until it becomes a linear function, thereby illustrating the "balancing" of wealth in the community. The central part of the curve represents the middle class in a community. The described model also "mathematizes" various wealth groups and precisely defines them.

1. INTRODUCTION

In studies of the inequality of the financial distribution of world wealth, the basic tools used are the Lorenz curve and the Gini index. The Lorenz curve was first introduced back in 1905 when the American researcher Max O. Lorenz devised a curve that represents a measure of the financial inequality of the distribution of wealth in a given community. The curve represents the functional relationship between the number of community members and their wealth. The Lorenz curve becomes a line (completely "straightens out") when wealth is balanced (eg 50% of people own 50% of the wealth). A few years later, in 1912, the Italian statistician and sociologist Corrado Gini defined an index that numerically, with a single number, indicates the level of inequality [1], [2], [3]. The Gini index is usually defined in terms of the Lorenz curve, but it can be defined in other ways. A community in which only one person owns all the wealth would correspond to a Gini index of 1, and a just society would correspond to a Gini index of 0. At the world level, the Gini index is currently around 0.6, depending on

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the data source [4]. The simplicity of a single number is the main value of the Gini index. On the other hand, the same index can be produced by different distributions, which can be considered a disadvantage. The Lorenz curve and the Gini index can be applied to many areas, financial and non-financial (ecology, health), where there is a lack of balance in some parameters. Indices can be applied to a specific industry, group of people, or even species in an ecology. References [5] and [6] describe financial applications. Other applications, [7], [8] and [9], describe education and healthcare models. There are other measures of inequality. such as Robin Hood index (Figure 2.2, also known as Hoover or Schutz index), Atkinson and Suits indices [10]. Our interest in this work is the development of (i) a mathematical model of the Lorenz curve and (ii) a recursive model of balancing wealth with social giving. The results indicate (iii) how far the world is from financial balance (156 years), as well as (iv) the importance of the middle class in balancing. The balancing algorithm shows how the middle class "pulls" wealth towards the balance. There is a huge imbalance between the rich and the poor in the world, in most human societies and countries, whether small or large. Furthermore, unfortunately, the recent events in world finance (the last crisis of 2008-2009 as well as the previous similar credit crises of the 1980s) testify to the growing rift between the richest and the poorest, and there is no visible action plan or even any desire of the strongest and richest to change this trend. In the annual world financial reports, Credit Suisse (and many other similar organizations, including the UN), [11]-[15], published numerous reports indicating a trend of fewer people owning more and more wealth. Reports show that less than 1% of the richest own more than 50% of the world's wealth. In this paper, we consider a mathematical model for community wealth, based on which wealth is balanced by social giving. This is a naive impractical assumption, but it is the first step in understanding how far a community is from balance in wealth. In our other works, we continue this model with the addition of investing. The ultimate goal of this research is to show that wealth is greater if investing and social giving are combined, compared to investing alone. Our interest is not only mathematical but to some extent also ideological, because we believe in a just world. In such a world, the Lorenz curve is a straight line (even distribution of wealth) and the Gini index is 0.

The paper is organized as follows. Section 2 provides a brief summary of the basics of the Lorenz curve and the Gini index, illustrated using data for several countries. Other indices are also mentioned, e.g. Robin Hood Index. Section 3 describes a new wealth distribution model, the "mean halved" (MH) wealth distribution model, which is based on a set of linear approximations from the richest to the poorest. The model is general and illustrated with examples. Section 4 deals with the specifics of the new model of wealth distribution, where it is assumed that all but the poorest group give a fixed percentage of their wealth to the community. As the wealth balancing algorithm progresses (per a given period of giving, for example annually), the initial unfavorable curve changes step by step, eventually leading to a perfect straight line. Various boundary conditions are satisfied as the algorithm progresses for each new dispensing cycle. At the end of Section 4, examples of the "balancing table" as well as the current wealth curve in the world, using the MH model presented in this paper, are given. The conclusion is in Section 5. At the end of the paper is a list of references.

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2. LORENZ CURVE AND INEQUALITY INDICES

The first part of the paper presents the Lorenz curve and gives several examples of countries and their curves. From a mathematical point of view, all curves are non-linear functions (wealth on the ordinate, number of community members on the abscissa, or vice versa). The "greater" the nonlinearity, the greater the difference in wealth between the poorest and the richest.

2.1. **Lorenz curve, Gini index**

Examples of Lorenz curves taken from different sources [12] are shown in Figure 2.1. All curves are only rough approximations. Our mathematical model is much more accurate. Although not visually equivalent to the standard Lorenz curve. the working model has the same information with the same line of balanced wealth. Figure 2.1 shows the Lorenz curves for Bangladesh, the UK, Brazil and the world. Interestingly, Bangladesh is ahead of Great Britain and Brazil, well ahead of the world average, but with less overall wealth. Figure 2.2 shows the general shape of the Lorenz curve and defines the Gini and Robin Hood indices. In our work, the shape of the Lorenz curve is adapted for the needs of precise mathematical analysis.

2.2. **Index of social giving**

Definition 2.1. *The social giving index "D" is the portion of the total wealth of the community (W_T) that can be given by all working community members for the common good of the poorer in the community:*

$$D = W_T/Z \tag{2.1}$$

Examples: Z=20, 40, 100, i.e. 5%, 2.5%, and 1% allowance. Index D can be constant or variable. Index D can be used to balance the wealth of the community, and a method to improve the economic situation. Our work presents the problem of inequality and balancing in a precise mathematical way, provides a method for explaining and assessing how far each community is from the ideal Lorenz line, and lays out the foundation for economic policies that reduce damaging economic booms and busts.

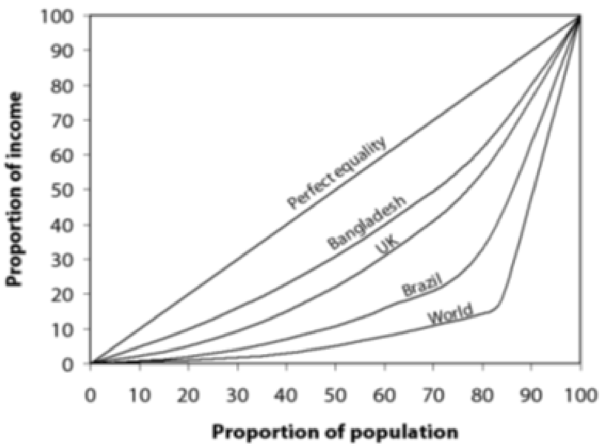

FIGURE 2.1. *Lorenz Curve Examples*

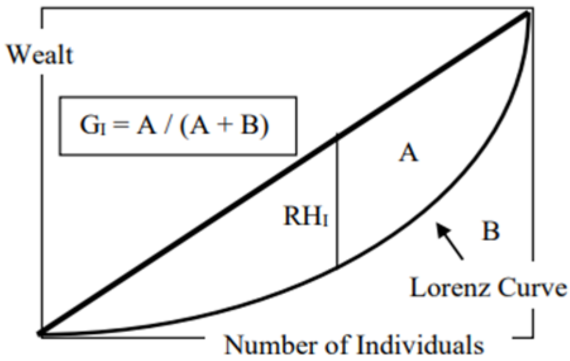

FIGURE 2.2. *Gini and Robin Hood indexes*

2.3. **Wealth Investment Index**

Our research is focused on how to combine social giving and wealth investing to achieve individual financial goals and social equity. Similar to the giving index D, we have:

Definition 2.2. *The wealth investment index "U" is the part of wealth W_T that can be invested:*

$$U = W_T/V \tag{2.2}$$

where V is the investment ratio. E.g. V=20, 40, 100 corresponding to 5%, 2.5%, 1% investment. In future work, we show that the combination of giving and investing can lead to very positive economic strategies, i.e. (i) increasing the total wealth of W_T and (ii) reducing the total risk of R_T:

$$W_T(investing + giving) > W_T(investing) \tag{2.3}$$

$$R_T(investing + giving) < R_T(investing) \tag{2.4}$$

3. WEALTH DISTRIBUTION MODEL

In this paper, we present a new wealth model that can be used for a variety of applications, from wealth balancing, to modeling constant (normalized) and variable community wealth, including analysis of the ”distance” of wealth from the ideal Lorenz line.

3.1. **Basic assumptions of the model**

We will now describe a simple wealth distribution model in which we assume a q-quantile distribution of wealth with q = 2, and apply it repeatedly to the desired level of wealth granularity. Simply put, we divide the total wealth W_T of the community into two halves, i.e. we find the middle point, as the initial distribution of wealth. We then divide one of the halves representing the poorer end of the wealth spectrum into two halves (two quarters of the original half) and so on, cutting the poorer end of the wealth in half. We assume that the distribution of wealth within each group is uniform. It can be generalized.

Definition 3.1. *The distribution model of the "mean halved" (MH) total wealth of W_T is:*

$$\begin{aligned} W_T &= W_T/2 + W_T/2 \\ &= W_T/2 + W_T/4 + W_T/4 \\ &= W_T/2 + W_T/4 + W_T/8 + W_T/8 \\ &= W_T(1/2 + 1/4 + 1/8 + 1/16 + \ldots + 1/(2L) + 1/(2L)) \end{aligned} \tag{3.1}$$

where L+1 is the total number of groups in the given community. This can also be written as:

$$W_T = W1 + W2 + \ldots + W_L + W_{L+1} = \sum W_m, m = 1, 2, ..., L+1 \tag{3.2}$$

with the i-th group:

$$W_i = W_T/2^i = \sum W_n, i = 1, 2, ..., L; n = i+1, ..., L+1 \tag{3.3}$$

that is, each previous wealth is the sum of the remaining wealth, with the boundary condition $W_{L+1} = W_L$. In addition, the total number of community members can be divided into the corresponding sum:

$$N_T = N_1 + N_2 + \ldots + N_{L+1} = \sum N_m, m = 1, 2, ..., L+1 \tag{3.4}$$

The last group W_{L+1} can be further divided into two smaller groups as desired. Finally, we come to the point in equation 3.3 when we decide that W_{L+1} is the poorest group, for practical and mathematical reasons. The last N_{L+1} corresponds to the last W_{L+1}. Additionally, $N_L \neq N_{L+1}$, with $N_{L+1} > N_L$ for most practical cases. For the whole world, for the poorest individual, L is 31, for about 3.4 billion working adults in 2023 [16], which is between 231 and 232. For a large city of about 1.05 million working people, L = 20. Practically we are not going to the level of an individual, but to the level of a large group of individuals. Through several examples we came to L = 6 as a practical number, so W_7 is the poorest group. Our MH model is very flexible and can be adjusted to any desired level of precision. Our main goal here is to use the wealth model for:

(1) Defining recursion for social giving and determining when group wealth balance is achieved. In further works we will describe:
(2) Defining a giving index, constant, variable, or sectoral, as a measure of the group's contribution
(3) A variable group wealth model that includes social giving as well as wealth investment to understand their relationship and changes in wealth between two or more periods of giving and investment
(4) Analysis of sensitivity, robustness and risk of the wealth model
(5) Demonstrate the beneficial effect of combined social giving and investment.

This is the key result of our research project, the first step of which is described in this paper.

Figure 3.1 shows the (non-linear) distribution of wealth approximated with linear segments. with the property of "halved middle" based on equations 3.1 - 3.4. In our model, we plot individual wealth along the vertical axis, not total wealth as in the standard Lorenz curve. Total wealth in our model is the area under the line segments in Figure 3.1. This is done to clearly define the balancing algorithm in Section 4. The horizontal axes show the number of community members (poorest on the right, richest on the left) divided into groups, so that each area corresponds to the total wealth in that group, according to equation 3.3. The size of the groups is not in the precise relationship in Figure 3.1.

The shape of the distribution function is general and can be applied to any practical situation. For example, if Figure 3 represents the world wealth, then w0 = 212 billion dollars is the current wealth of Frenchman Bernard Arnault (not Elon Musk anymore!) [16], and N1 $<$ 1% is the number of the richest people in the world. The value of w1 can be calculated using a simple geometric calculation from Figure 3, given w0 and W1. The variable w1 represents the smallest amount of wealth in the W1 group. If the 1-line model is not valid, then the 2- or 3-line model is modified (Section 3.4). Values along the axes can be normalized to 1 or 100 to work with percentages. Normalized values to 100 are defined as:

$$\text{Individual wealth}: w_n or = 100(w/w0),$$
$$\text{Number of community members}: n_n or = 100(n/nL+1). \quad (3.5)$$

3.2. Description of the basic model

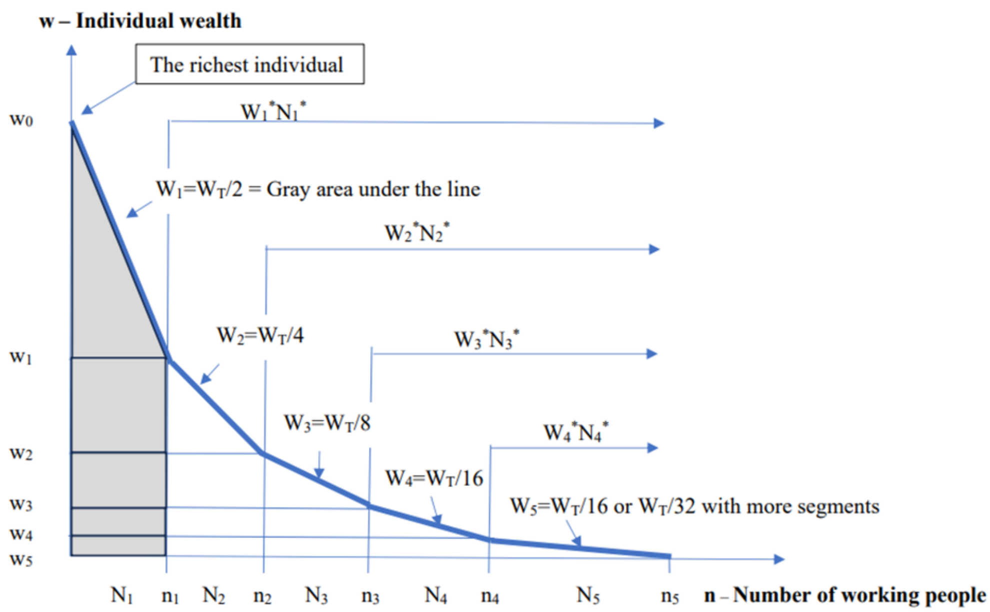

FIGURE 3.1. *MH Wealth Distribution Model*

The total wealth in Figure 3.1 is the area under the line segments. We start with group W_1. The wealth of that group is equal to the area under the first line

$$W_1 = W_T/2 = n1w1 + (n1/2)(w0 - w1) = N1(w1 + w0)/2 \quad (3.6)$$

where $N1 = n1 - n0$, $n0 = 0$. Also, W_1 is the sum of all individual wealth in the group:

$$W_1 = W_T/2 = \sum w1^m \tag{3.7}$$

where m = 1,2,...,N1 , $w1^1$ = w0 , w1 = $w1^{N1}$. This represents the sum of all the individual wealths $(1xw1^m)$ Figure 3.1. The next wealth W_2 is calculated as:

$$W_2 = W_T/4 = (n2 - n1)w2 + (n2 - n1)(w1 - w2)/2 = N2(w2 + w1)/2 \tag{3.8}$$

where $N_2 = n_2 - n_1$. As in 3.8 we have:

$$W_2 = W_T/4 = \sum w2^m \tag{3.9}$$

and m = 1,2,...,N_2 i $w_2^1 = w_1$, $w_2 = w_1^{N2}$. Using induction we have a general result for W_i:

Result 3.1. Wealth W_i is the vertical area under the corresponding linear segment in Figure 3.1:

$$W_i = W_T/2^i = (n_i – n_{i-1})(w_i + w_{i-1})/2 = N_i(w_i + w_{i-1})/2 \tag{3.10}$$

$$N_i = n_i – n_{i-1}, n_0 = 0 \tag{3.11}$$

where N_i represents the number of members of the group W_i. Each W_i, $i = 1, 2, 3, \ldots,$ $L+1$ also represents the corresponding sum of individual wealths:

$$W_i = W_T/2^i = \sum wi^m \tag{3.12}$$

$$wi^1 = w_i, w_i = wi^{Ni}, m = 1, 2, \ldots, N_i \tag{3.13}$$

From 3.13 we obtain:

$$W_i = W_{i-1}/2 \tag{3.14}$$

and combined with 3.12 we get:

$$Wi_1 = W_T/2^{i-1} = N_i(wi + w_{i-1}) \tag{3.15}$$

The above equations are used when normalizing recursive formulas. Plus, W_T can also be expressed as integrals:

$$W_T = \int W(n)dn = \int N(w)dw \tag{3.16}$$

$$= \sum W_i = \sum\sum wi^m, i = 1, 2, \ldots, L+1, m = 1, 2, \ldots, N_i. \tag{3.17}$$

The limits of the first integral are w_{L+1} (practically very close to 0) to w0, and of the second are from 0 do n_{L+1}. The two functions W(n) and N(w) are inverses of each other, and the two integrals can be calculated by some numerical methods starting from $\sum W_i$ u 3.17. From Figure 3.1 we see that the function W(n) consists of individual lines:

$$W_i(n) = K_i(n + ni*) \tag{3.18}$$

where K_i, is line W_i(n) slope:

$$K_i = (w_i – w_{i-1})/(n_i – n_{i-1}) = w_i^*/(n_i – n_{i-1}) \tag{3.19}$$

$$w_i^* = (w_i n_i - 1 – w_{i-1} w_i)/(w_i – w_{i-1}) \tag{3.20}$$

and w_i^* is the point of intersection of the vertical axes, given W_T, w_0 and N_i while w_i is calculated recursively from:

$$w_i = (2W_i)/N_i – w_{i-1} = (W_T/2^{i-1})/N_i – w_{i-1}. \quad (3.21)$$

The corresponding inverse functions N_i(w) can be found by solving 3.18 for w as a function of n. The above calculations are used in future works where we calculate the number of community members, i.e. balancing the horizontal axis in Figure 3.1, after wealth has been balanced (Chapter 4). These two balancing operations form the basis for the wealth normalization process. In the normalized case, the ideal Lorenz line corresponds to the function W(n):

$$W(n) = -Kn + w^* = -n + 100 \quad (3.22)$$

$$W_i(n) = K_i n + N_i^* = -n + N_i^* \quad (3.23)$$

where K_i = - 1, i = 1,2,..., L+1 represents the slope of the local Lorenz line, which allows us to accurately categorize different groups into terms of rich, middle class and poor (or even more precisely).

3.3. **Definition of wealth classes**

From Definition 2.2 and the equations above, we can obtain a precise mathematical definition of wealth class. We formally define different classes as one or more groups with a certain relationship to the ideal Lorenz line. Using 3.22 i 3.23 above, we have:

Definition 3.2. *The classes of wealth in the MH model of wealth distribution are:*

$$\begin{aligned} &\textit{Rich Classes: } |K_i| = |K_R| >> 1 \\ &\textit{Middle Classes: } |K_i| = |K_M|, |K_P| < |K_M| < |K_R| \\ &\textit{Poor Classes: } |K_i| = |K_P| << 1 \end{aligned} \quad (3.24)$$

where "i" denotes some range of values, in each of the above 3 general wealth groups. Furthermore, we can divide the middle class into three separate groups:

$$\begin{aligned} &\textit{Upper Middle Class:} 1 < |K_M| < |K_R| \\ &\textit{"Middle" Middle Class: } |K_M| \approx 1 \ \ (\textit{closest to the ideal value of 1}) \\ &\textit{Lower Middle Class: } |K_P| < |_K M| < 1 \end{aligned} \quad (3.25)$$

In this paper, after testing several examples, we selected 7 *groups Wi with* $i = 1,2,...,L+1$, $L = 6$.

The importance of the middle class lies in the fact that it is the closest to the ideal Lorenz line. It is known from economic theory that the more numerous the middle class is, the better it is for the community. We also divided the rich and poor classes into two groups. With L = 6, the MH wealth distribution model can be precisely defined as:

Definition 3.3. *Wealth groups W_i, i=1,2,...,7, i=1,2,...,7, in the MH model are 100% in total, i.e.:*

$W1$ – Super rich group (50% of wealth)
$W2$ – Very rich group (25% of wealth)
$W3$ – Upper middle class group (12,5% of wealth)
$W4$ – Middle middle class group (6,25% of wealth)

W5 – Lower middle class group (3,125% of wealth)
W6 – Poor group (1,5625% of wealth)
W7 – VVery poor group (1,5625% of wealth) (3.26)

with the corresponding number of group members N_i and line slopes K_i and $i = 1, 2, \dots, 6, 7$.

A few comments are in order:

(1) The super-rich group can be further divided into $W_1 = W_1^1 + W_1^2$, where W_1^1 is a few thousand extremely rich individuals, who own about 16.7% of the world's total wealth, and W_1^2 is a very rich group with 1% of the world's population who owns 1/ 3 or 33.3%.
(2) Rich W_2 is a "transient" group, give as much as you get (see Section 4)
(3) When all the W_3, W_4 and W_5 middle class percentages are added together, we get 12.5% + 6.25%+ 3.125% = 21.875% which agrees very well with the UN estimate of middle class wealth at around 22%. This may be a coincidence, or an indication of the general validity of our wealth model.

By combining Definitions 3.2 i 3.3, with Figure 5.1 (which will be further explained below), we get:

Result 3.2. Coefficients of Lorenz lines and W groups (as of 2016, updated for 2024):

$W1$ – "Super Rich" group: $|K_1| = 50$
$W2$ – "Rich" group: $|K_2| = 10$
$W3$ – "Upper Middle Class" group: $|K_3| = 3$
$W4$ – Middle "Middle Class" group: $|K_4| = 1,08$
$W5$ – "Lower Middle Class" group: $|K_5| = 0,45$
$W6$ – "Poor group": $|K_6| = 0,043$
$W7$ – "Very Poor" group: $|K_7| = 0,028$ (3.27)

with $|K4|$ = 1,08 for W4 as the closest to the ideal Lorenz line $|K|$ = 1, per Definition 3.1.

3.4. **Modified linear model**

If the calculation in 3.21 does not give a reasonable value for wi, it implies that the 1-line Wi model in Figure 3.1 should be corrected. If we look at group W1, less than 1% of individuals own more than 50% of the wealth. When we made the first drafts of our model in 2016, the wealth situation was a little "better", namely about 1% owned about 50%. If we calculate w1 from 3.21, we get a negative value. Therefore, the W_1 group should be broken down into several lines (the same area under W_1 is kept) to calculate w1. Figure 4.1 shows any W_i modeled with 2 - line segments instead of one.

3.4.1. ***Linearity test.*** To check the validity of the 1-line model we need a linearity test. First, w_i is calculated from equation 3.21. If the value is positive (and economically "reasonable"), the linearized model is valid. Otherwise w_i is broken into multiple linear segments as shown in Figure 4.2, with two linear segments. If necessary, the process is repeated. With or without the linearity test, the balancing algorithm of Section 4 along the "w" axis holds either way because W_i, $W_T/2^i$, with one or more lines.For balancing along the "n" axis, we must satisfy the boundary conditions between the groups $(n_1, N_1, w_1; n_2, N_2, w_2; ...,)$. Details are given in our future works.

4. RECURSIVE WEALTH BALANCING MODEL WITH SOCIAL GIVING

In this paper, we present several reasonable and practical assumptions regarding social giving and wealth balancing. Due to limitations in the size of the text, this paper deals only with the MH model and the mathematics of social giving. Investment, as another key part of the model, is described in other texts.

4.1. Basic assumptions

(1) During the cycle of giving and investing (one year) the total wealth of W_T is either constant (normalized to 100) or variable, which can also be normalized in each cycle.
(2) The poorest members of the community do not contribute to social giving. This is the W_i group that "does not give". This assumption is logical and fair in practice, and is often practiced or at least recommended in various world religions. On the other side of the model, the super rich group only gives to the lower classes. These assumptions are reflected in our mathematical model.
(3) Social giving is evenly distributed in the community and reaches all corners of the community, with contributions from the rich to the poor, along defined wealth groups. The richest group only gives, the other groups give and receive.
(4) Before each new cycle, we assume that total wealth is either (i) normalized or (ii) not normalized. Normalization can be done in several ways, as discussed below. This can include new wealth generated between two cycles (say through investing)
(5) We assume that each member of the community (except the poorest group) gives an equal share (percentage) of his wealth, Z, which is distributed for the common good, equally throughout the community. Our model allows variable giving Z as well as "sectoral" giving say for agriculture or other areas of interest.
(6) Individual groups can invest an arbitrary part of their wealth according to their needs and wishes.

4.2. Normalized wealth notation

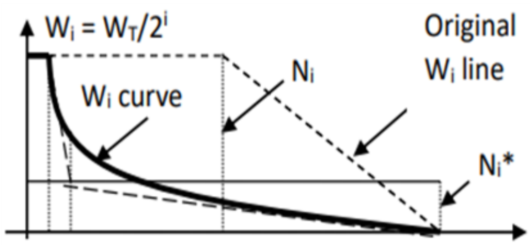

FIGURE 4.1. *W_i with 2-line approximation*

With the assumptions from Section 3, we continue with the description of the recursive wealth balancing algorithm. We introduce the normalized dynamical notation for the group W_i from Figure 3.1: for the kth cycle, and 1/2 part of the wealth in equation (6). Figure 4.2 showing the normalized W_T, divided into mean halved groups. The notation on the right-hand side of (32) is a bit complicated, but serves the purpose of explaining the algorithm at this point. We will return to a simpler notation shortly.

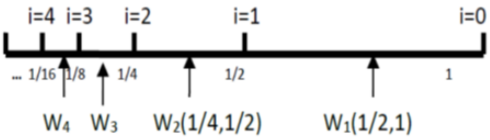

FIGURE 4.2. *Wealth normalization*

The initial condition before any social giving corresponds to k = 1, and for the richest half of the wealth, i.e. i = 1, we have W1 in Figure 3 which corresponds to W1(k)=Wk(1/2,1) in (32) notation. This corresponds to the richest half of wealth from 1/2 to 1 in normalized terms in Figure 4.2 when total normalized wealth is:

$$W_i = W_i(k) = W_k(1/2^i, 1/2^{i-1}) \tag{4.1}$$

for the kth cycle, and 1/2 part of the wealth in equation 3.2. Figure 4.2 showing the normalized W_T, divided into mean halved groups. The notation on the right-hand side of 4.1 s a bit complicated, but serves the purpose of explaining the algorithm at this point. We will return to a simpler notation shortly. The initial condition before any social giving corresponds to k = 1, and for the richest half of the wealth, i.e. i = 1, we have W_1 in Figure 3 which corresponds to W_1(k)=W_k(1/2,1) in 4.1 notation. This corresponds to the richest half of wealth from 1/2 to 1 in normalized terms in Figure 4.2 when total normalized wealth is:

$$W_T/W_T = 1 = \sum W_i/W_T. \tag{4.2}$$

For i = 2, we have W_2(k)=W_k(1/4,1/2) which corresponds to the first quarter of the wealth remaining from W_k(1/2,1). When i = 3 we have W_k(1/8,1/4) which corresponds to the first 8^{th} of the remaining wealth of W_k(1/4,1/2), etc. We also note that after each normalization step , for any k:

$$\begin{aligned} W_k(1/2,1) &= W_1(k) = 1/2 \\ W_k(0,1/2) &= W_1^*(k) = 1 - W_1(k) = 1/2 \end{aligned} \tag{4.3}$$

where W_1(k) represents 50% of the total wealth of the community owned by the richer, and the complementary W_1^*(k) is 50% owned by the other less rich and poorer (see Figure 3.1 for W_1^*(k)). We then divide W_1^*(k) into two equal quarters representing the two quarters of the poorest 50%:

$$\begin{aligned} W_k(1/4,1/2) &= W_2(k) = 1/4 \\ W_k(0,1/4) &= W_2^*(k) = W_1^*(k) - W_2(k) = 1/4 \end{aligned} \tag{4.4}$$

followed by dividing W_2^*(k) into equal 8s representing the two 8ths of the poorest 25%:

$$\begin{aligned} W_k(1/8,1/4) &= W_3(k) = 1/8 \\ W_k(0,1/8) &= W_3^*(k) = W_2^*(k) - W_3(k) = 1/8 \end{aligned} \tag{4.5}$$

etc., through the general i-th group W_i:

$$W_k(1/2^i, 1/2^{i-1}) = W_i(k) = 1/2^i$$
$$W_k(0, 1/2^i) = W_i^*(k) = W_{i-1}^*(k) - W_i(k) = 1/2^i \tag{4.6}$$

The final step is to divide the last group into two equal parts:

$$W_k(1/2^{L+1}, 1/2^L) = W_{L+1}(k) = 1/2^{L+1}$$
$$W_k(0, 1/2^L) = W_L^*(k) = W_{L-1}^*(k) – W_L(k) = 1/2^L \tag{4.7}$$

where $W_L^*(k)=W_{L+1}(k)$ is the last group that does not give, its members only receive from other groups. Each group in 4.1-4.7, i.e. W_1, W_2, …, W_L (richer halves) gives a fixed proportion to their complementary pairs W_1^*, W_2^*, …, W_L^* (poorer halves), while the first group W1 only gives and receives nothing. As for the W_i^*s, those "complementary" groups also give and receive, except for the last $W_L^* = W_{L+1}$ which only receives. This holds under a fixed total wealth and fixed giving assumption. New wealth produced (or lost) between giving periods can easily be incorporated into our model, normalized or not. Our future work elaborates a variable giving index. We also note that in the normalized case $W_i(k) = W_i^*(k)$, i=1,2,…,L, the right and left sides of the center 2^i in Figure 4.2, plus $W_L^*(k) = W_{L+1}(k)$. If normalization is not performed, this is no longer the case as the wealth values of the groups change. In essence, total wealth is normalized if the assumptions in Definition 1 hold. More on that below.

4.3. **Algorithm for balancing the MH wealth model**

The recursive social giving algorithm starts with $W_k(1/2,1)$, the richest 50% of the total wealth of W_T. Giving starts here. We assume in the kth period that a fixed percentage Z of $W_k(1/2, 1)$ wealth is given uniformly across the poorer group $W_k(0, 1/2)$. In practical terms this can be given to different social needs across $W_k(0, 1/2)$. We have two situations: (i) k-iteration without wealth normalization, i.e. we do not fit $W_i(k)$ to Definition 1 for each k. (ii) With normalization, all k (Figure 3.1) the groups $W_i(k)$ and Ni are recalculated per Definition 1. In both cases we use the same simplified notation $W_i(k)$.With normalization, we get very interesting (perhaps fundamental) relationships between wealth groups.

4.3.1. ***Model of social giving without normalization.*** This corresponds to n_i (N_i) and w_i fixed in Figure 3.1. The following equations describe the give/receive in the (k+1)-th period as a function of the previous kth cycle. Here we switch to simplified notation:

$$W_1(k+1) = W_1(k) – W_1(k)/Z = (Z-1)W_1(k)/Z$$
$$W_1^*(k+1) = W_1^*(k) + W_1(k)/Z. \tag{4.8}$$

Then the poor portion of 50% of $W_1(k)$ is shifted and divided into two quarters $W_2(k)$ and $W_2^*(k)$, where a quarter of $W_2(k)$ gives and a quarter of $W_2^*(k)$ receives from both $W_1(k)$ and $W_2(k)$. The contribution of $W_1(k)$ is equally divided between two quarters of W_2(k) and $W_2^*(k)$, so $W_2^*(k)$ receives only half of W_1(k)/Z and full W_2(k)/Z:

$$W_2(k+1) = W_2(k) + W_1(k)/(2Z) – W_2(k)/Z \tag{4.9}$$

$$W_2^*(k+1) = W_2^*(k) + W_1(k)/(2Z) + W_2(k)/Z. \tag{4.10}$$

Here W_1(k) decreases, W_1^*(k),W_2(k),W_2^*(k) increase. We continue in the same way and using induction we get:

Result 4.1. The recursive wealth of the i-th group W_i(k) in the non-normalized balancing model of the "mean halved" MH wealth model with the assumption of a constant endowment index is:

$$W_i(k+1) = (Z-1)W_i(k)/Z + \sum W_{i-n}(k)/(2^n Z) \tag{4.11}$$

$$W_i^*(k+1) = W_i^*(k) + \sum W_{i-m}(k)/(2^m Z) \tag{4.12}$$

$$W_L^*(k) = W_{L+1}(k+1) \tag{4.13}$$

$$W_T = \sum W_p(k) + W_i^*(k) \tag{4.14}$$

with n = 1,2,. . . ,i-1, and m = 0,1,. . . ,i-1 i p = 1,2,. . . ,i. The algorithm goes to i = L, when we have a final giving from W_L(k) and receiving at W_L^*(k) = W_{L+1}(k).

The equations in Result 4.1 can be simplified with the notation:

$$W_i(k+1) = (1-A_Z)W_i(k) + A_Z \sum B_{in} W_n(k) \tag{4.15}$$

$$W_i^*(k+1) = W_i^*(k) + A_Z \sum B_{im} W_m(k) \tag{4.16}$$

where $A_Z = 1/Z, B_{in} = 1/2^{i-n}$, with $n = 1,2,\ldots,i-1$, $m = 0,1,\ldots,i-1$, $i = 1,2,\ldots,L$. Then, a new cycle of giving begins and we go to $k+2$, etc., until the Lorentz curve is an ideal line when a uniform distribution of wealth is achieved. This happens in the ideal case when K_i u 3.22 has the same value for all i.

Tests for achieving balance are in Section 4.6.1. The balancing algorithm stops at this point. Before renormalization, equations 4.2 -4.6 applied to the iterated wealth values do not hold. Not yet, not until the new renormalization is complete, as described in the next section. The expression $\sum B_{in} W_n(k)$, $n = 1,2,\ldots,i-1$, in Result 4.1 in the normalized case for $W_i(k) = W_T/2_i$ reduces to:

$$\begin{aligned}\sum B_{in} W_n(k) &= W_1(k)/2^{i-1} + W_2(k)/2^{i-2} + \ldots W_i - 1(k)/2^i \\ &= W_T/2^i + W_T/2^i + \ldots + W_T/2^i = (i-1)W_i(k).\end{aligned} \tag{4.17}$$

The mentioned equation is very important, because it connects the general and normalized case.

4.3.2. ***Model of social giving with normalization.*** We proceed with further simplifications of the above equations, when normalization is performed between each k and k+1 administration cycle. This case corresponds to the variables n_i (N_i) and w_i. As in the non-normalized case 4.3.1, we start from the same equations:

$$W_1(k+1) = W_1(k)–W_1(k)/Z = (Z-1)W_1(k)/Z \quad (4.18)$$

$$W_1^*(k+1) = W_1^*(k) + W_1(k)/Z = (Z+1)W_1(k)/Z \quad (4.19)$$

where we used the fact that the two terms W_1(k) and W_1^*(k) are equal when normalization is performed. Then we use Definition 3.1 to calculate:

$$\begin{aligned} W_2(k+1) &= W_2(k) + W_1(k)/(2Z)–W_2(k)/Z \\ &= W_2(k) + W_2(k)/Z–W_2(k)/Z = W_2(k) \quad (4.20) \end{aligned}$$

$$\begin{aligned} W_2 * (k+1) &= W_2^*(k) + W_1(k)/(2Z) + W_2(k)/Z \\ &= W_2 * (k) + W_2(k)/Z + W_2(k)/Z \\ &= W_2^*(k) + W_2^*(k)/Z + W_2^*(k)/Z \\ &= (Z+2)W_2^*(k)/Z. \quad (4.21) \end{aligned}$$

We see that $W_2(k+1) = W_2(k)$, and that the W_2 group gives the same as it receives ("passive group"). For W_3 we have: We see that W_2(k+1) = W_2(k), and that the W_2 group gives the same as it receives ("passive group"). For W_3 we have::

$$\begin{aligned} W_3(k+1) &= W_3(k)–W_3(k)/Z + W_1(k)/(4Z) + W_2(k)/(2Z) \\ &= W_3(k)–W_3(k)/Z + W_3(k)/Z + W_3(k)/Z = W_3(k) + W_3(k)/Z \quad (4.22) \end{aligned}$$

Complementary wealth W_3^* is given below:

$$\begin{aligned} W_3^*(k+1) &= W_3^*(k) + W_1(k)/(4Z) + W_2(k)/(2Z) + W_3(k)/Z \\ &= W_3^*(k) + W_3(k)/Z + W_3(k)/Z + W_3(k)/Z \\ &= W_3^*(k) + W_3^*(k)/Z + W_3^*(k)/Z + W_3^*(k)/Z = (Z+3)W_3^*(k)/Z. \quad (4.23) \end{aligned}$$

We continue in the same way and using induction, with A_Z = 1/Z, we conclude the following general result for the i-th Wi group with normalization between k and k+1 cycles:

Result 4.2. The recursive wealth of the i-th group in the normalized wealth balancing model with a "mean halved" (MH) is:

$$W_i(k+1) = [1 + (i-2)A_Z]W_i(k) \quad (4.24)$$

$$W_i^*(k+1) = (1 + iA_Z)W_i^*(k) \quad (4.25)$$

(a special case of Result 4.1), plus:

$$W_L^*(k) = W_{L+1}(k+1), i = 1, 2, \ldots, L. \quad (4.26)$$

The algorithm terminates at i = L, with the final step giving $W_L(k)$ do $W_L^*(k) = W_L(k+1)$. At this point the algorithm stops. Before continuing for k+1, wealth normalization is done using the corrected values at k. Equations 4.2 - 4.6 hold. Figure 4.3 summarizes the process. Thick arrows indicate giving and block arrows indicate equal halving. The upper shaded block (richest half) only gives and the lower shaded block (poorest group) only receives. Between iteration steps, normalization (remodeling) is performed so that the next iteration of balancing starts with normalized Figure 3, with new iterated values of $w_i, n_i andW_i$. Normalization can be done analytically or numerically for the integration of N(w) or W(n), and by dividing the area under any function in groups, with $W_T = W_T(1/2 + 1/4 + 1/8 + \ldots + 1/(2L) + 1/(2L))$. Finally, normalization of W_T i N_T to 1 or 100 follows.

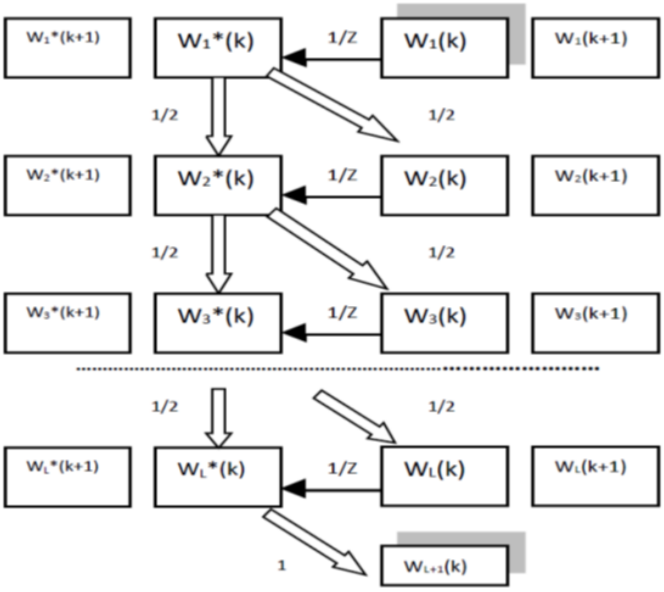

FIGURE 4.3. *MH wealth model balancing process*

4.3.3. ***Verification of total wealth.*** It should be checked whether the total wealth of W_T is preserved after the balancing algorithm (algorithm integrity check). The sum of all corrected (i =1,2,...,L+1) wealth values should be equal to the initial W_T so that the balancing algorithm is consistent. For the normalized case, the check confirms the consistency:

$$\begin{aligned}
W_T &= \sum_i W_i(k+1) + W_{L+1}(k+1) \\
&= \sum_i \{[Z+(i-2)]/Z\} W_i(k) + W_L^*(k) \\
&= (1/Z)[(Z-1)W_1(k) + (Z)W_2(k) + (Z+1)W_3(k) + \ldots \\
&\quad + (Z+L-2)W_L(k) + (Z+L)W_L(k) \\
&= (1/Z)[(Z-1)W_1(k) + (Z/2)W_1(k) + (Z+1)W_1(k)/4 + \ldots \\
&\quad + (Z+L-2)W_1(k)/2L - 1 + (Z+L)W_1(k)/2^{L-1}] \\
&= (1/Z)W_1(k)[(Z-1) + (Z/2) + (Z+1)/4 + \ldots \\
&\quad + (Z+L-2)/2^{L-1} + (Z+L)/2^{L-1}] \\
&= W_1(k)[(Z/Z)(1 + 1/2 + 1/4 + \ldots + 1/2L + 1/2L) \\
&\quad + (1/Z)(-1 + 1/4 + 2/8 + 3/16 + 4/32 + \ldots + (L-2)/2^{L-1} + L/2^{L-1})] \\
&= 2W_1(k) + (1/Z)(0) = W_T. \qquad (4.27)
\end{aligned}$$

4.4. **Average wealth received**

As the Giving Index causes wealth to be balanced in different groups, these groups give and receive additional wealth according to Outcomes 4.1 and 4.2. We need to calculate what the averages are for each individual in each group so that we can recalculate and renormalize group richness as well as the number of group members. From the previous results we get the following: **Result 4.3** The total received net wealth (received minus given) for the group $W_i(k)$, with Ni members is:

(1) General non-normalized case:

$$\Delta W_i^T(k) = A_Z[\sum B_{im}W_m(k) - W_i(k)] \tag{4.28}$$

(2) Normalized case:

$$\begin{aligned}\Delta W_i^T(k) &= A_Z[\sum_m B_{im}W_m(k) - W_i(k)]\\ &= A_Z[(i–1)W_i(k) - W_i(k)] = A_Z(i–2)W_i(k)\end{aligned} \tag{4.29}$$

(3) The average net wealth per wealth group member and the total average wealth per N_i members are:

$$\Delta W_i^A(k) = \Delta W_i^T(k)/N_i, W_i^A(k) = W_i(k+1)/N_i,\ i = 1,2,\ldots,L \text{ i } m = 1,2,\ldots,i-1 \tag{4.30}$$

For the last poorest group $W_L + 1$ we have a general and normalized case for average wealth:

$$\begin{aligned}\Delta W_{L+1}^T(k) &= A_Z[\sum B_{(L+1)m}W_m(k)]\\ \Delta W_{L+1}^T(k) &= A_Z[(L+1–1)W_{L+1}(k) = LA_ZW_L + 1(k)\end{aligned} \tag{4.31}$$

4.5. **Normalization**

To understand the normalization process, we assume that in some period k all groups W_i(k) are normalized and a new (k+1) giving cycle begins. All newly calculated W_i(k+1) are denormalized as a result of their giving and receiving.

4.5.1. ***Denormalization of wealth.*** Then, to calculate some group's wealth values for the (k+1)th period, we recall that W_1(k) (normalized over period k) gives W_1(k)/Z over N_1^*,which means giving all groups starting from W_2 to W_{L+1} uniformly. Due to this assignment, W_1 is denormalized and is equal to $W_1(k+1)$. Similarly, $W_2(k)$ receives its part W_1(k)/(2Z) = W_T/(4Z) from the normalized W_1(k) and gives its part W_2(k)/Z = W_T/(4Z) to W_3 through W_{L+1}. So the two amounts are equal, and W_2(k+1) remains normalized. The other half of W_1(k)/(2Z) of the total W_1 given W_1(k)/Z distributes through W_3 through W_{L+1}. Other groups for i > 2 receive more than they give and are thus denormalized. Group W_2 is a special group, because it gives the same as it receives, and on one side is W1, which reduces its wealth due to giving, and groups W_3 - W_{L+1} increase its wealth due to overall giving and receiving. Once the process is complete for all i = 1,2,..., L+1, we need to go back to W1(k+1) and see how to normalize it again. This is important to understand because of the elegant and simple properties of normalized groups. The term W_1(k)/Z (not just W_1(k)/(2Z)) must be borrowed from the neighboring group W_2(k+1) to compensate for giving W_1(k). This compensation comes from the members in group W2 that need to be moved in the renormalization from the still normalized W_2(k+1) to the denormalized W_1(k+1). But, due to member wealth borrowing from W_2 to re-normalize W_1, W_2 is also denormalized. This process moves through the group hierarchy.

4.5.2. ***Re-normalization of wealth.*** We now analyze the renormalization of each W_i group. The other side of renormalization, that of the number of members of each group, is dealt with in another paper. From the above we can write for W_1(k+1) the renormalizing term:

$$\Delta W_1^C(k) = W_1(k)/Z = A_Z W_1(k) = W_T/(2Z). \tag{4.32}$$

This amount is borrowed from $W_2(k+1)$ and moved to W1(k+1) to normalize it. This obviously denormalizes $W_2(k+1)$, so the same amount must be borrowed from $W_3(k+1)$ to renormalize $W_2(k+1)$. So we have to consider what is net received in W_3. From Section 4.4 and Result 2.1, for W_3 we can write:

$$\begin{aligned}\Delta W_3^T(k) &= A_Z[\sum_m B_{3m} W_m(k) – W_3(k)] \\ &= A_Z[(3-1)W_3(k) - W_3(k)] = A_Z(3–2)W_3(k).\end{aligned} \tag{4.33}$$

From the above amount one needs to subtract $\Delta W_1^C(k)$ to give to $W_2(k+1)$, i.e:

$$\begin{aligned}\Delta W_3^T(k) - \Delta W_1^C(k) &= A_Z(3–2)W_3(k) - A_Z W_1(k) \\ &= A_Z[W_3(k) – 4W_3(k)] = -3A_Z W_3(k)\end{aligned} \tag{4.34}$$

where we used $W_1(k) = 2W_2(k) = 4W_3(k)$ for normalized wealth groups. Hence, we can further write modified $W_3^M(k+1)$ as in:

$$W_3^M(k+1) = W_3(k) + \Delta W_3^T(k) - \Delta W_1^C(k) = W_3(k) - 3A_Z W_3(k) \tag{4.35}$$

where $W_3(k)$ is normalized. The next step is to normalize $W_3^M(k+1)$ taking $3A_Z W_3(k)$ from $W_4(t+1)$. The above amount needs to reduced by $3A_Z W_3(k)$ to be given to $W_3^M(k+1)$, i.e.:

$$\begin{aligned}\Delta W_4^T(k) &= A_Z[\sum_m B_{4m} W_m(k) – W_4(k)] \\ &= A_Z[(4–1)W_4(k) – W_4(k)] = A_Z(4–2)W_4(k)\end{aligned} \tag{4.36}$$

where we used $W_3(k) = 2W_4(k)$ for normalized groups. Further we write modified $W_4^M(k+1)$ as:

$$\begin{aligned}\Delta W_4^T(k) - 3A_Z W_3(k) &= A_Z(4–2)W_4(k) - 3A_Z W_3(k) \\ &= A_Z[2W_4(k) – 6W_4(k)] = -4A_Z W_3(k).\end{aligned} \tag{4.37}$$

Hence, we can further write $W_4^M(k+1)$ as:

$$W_4^M(k+1) = W_4(k) + \Delta W_4^T(k) - 3A_Z W_4(k) = W_4(k) - 4A_Z W_4(k) \tag{4.38}$$

where $W_4(k)$ is normalized. The next step is the normalization of $W_4^M(k+1)$ borrowing $4A_Z W_4(k)$ from $W_5(t+1)$, etc. General results then are as follows:

Result 4.4. Renormalization of $W_i(k+1)$ requires adding to it the amount:

$$iA_Z W_i(k) \tag{4.39}$$

from $Wi+1(k+1)$, ending with i = L+1, until $W_{L+1}(k+1)$ which produces $LA_Z W_L(k)$ for $W_L(k+1)$ to be normalized again. This can be easily verified from Result 4.2 of Section 4.3.2. After re-normalization, all $W_i(k+1)$ are normalized again, and the same procedure is repeated for (k+2), (k+3), etc. Due to limited space, we do not deal with the renormalization of the number of group members. This is given in our other works.

4.6. **Wealth balancing tables**

Using the Results of this section we can generate balancing tables, each for a specific index of giving. This is illustrated by an example.

4.6.1. ***When is balance achieved?*** As the balancing algorithm progresses, k = 1, 2, 3, . . . , there is a point where the rich will become poor and the poor will become rich, and this is not the intention of the giving process (despite the fact that the poor might like it). Instead, the idea is to have a balanced distribution across all groups. Therefore, we must determine when to stop the recursion, which is when the ideal Lorenz line is reached, and the giving process should be stopped. In theory this is achieved when all segments in Figure 3.1 are at the same angle and K_i in 3.19 has the same value for all W_i. This corresponds to the average total W_{TA} wealth among N_T members as well as the average of each W_i:

$$W_{TA} = W_T/N_T = W_i(k+1)/N_i \tag{4.40}$$

Given the average wealth of W_{TA} we can calculate the giving period to stop the algorithm for each group W_i as:

$$W_i(k_B) = (W_T/N_T)N_i = W_{TA}N_i \tag{4.41}$$

where k_B is the wealth balancing period. This assumes that all N_i are known. If not, what are known as stopping criteria can be used. See examples in Section. 4.7. Another method to stop is:

$$|W_i(k+1) - W_i(k)|/N_i = |\Delta W_i(k+1)|/N_i = |W_i^A(k+1)| < \epsilon_W \tag{4.42}$$

for a small ϵ_W . Another option is to test the sums of the wealth against some given thresholds:

$$\sum W_i(k+1) < \delta W. \tag{4.43}$$

The range of values ϵ_W and δ_W is related to average values of wealth and can be determined by some numerical and statistical analysis. One of the potential problems with the above methods is that we may not achieve the true average wealth. Another simple test can be used to avoid this, as we illustrate in the examples in this section:

$$|W_i^A(k_{min})| > W_A > |W_i^A(k_{min}+1)|; k_{min} \leq k_B \leq k_{min}+1. \tag{4.44}$$

The algorithm can be stopped as soon as one or more of the above conditions are met. In the case of 4.44, we perform an additional calculation step to make sure that the algorithm has reached only one step above the threshold. In Section 4.7 we illustrate the stopping method 4.44 using several concrete examples.

4.6.2. ***Interpretation of the balance table.*** The numerical examples in this section demonstrate a number of important features of our wealth balancing algorithm. Tables carry universality as a theoretical and numerical tool. Here are the important points:

(1) All calculations are normalized to $W_T = 100$ and $N_T = 100$. These tables are universal normalized wealth balancing tables and apply to ANY N_i, given $\sum N_i = N_T$ and ANY W_i given $\sum W_i = W_T$, both normalized to 100, hence, the results can be interpreted as percentages.
(2) Each table for a given giving index Z starts with the same set of numbers (ideally 50%, 25%, 12.5%, 6.25%, etc., but the algorithm is very robust and it works for other numbers as well). These numbers indicate the percentages of groups from which balancing starts. At this time we do not state anything about how many people own 50% or 25% etc.
(3) The examples illustrate L = 6. Recall that the (L+1) group is the group that does not give (the poorest), and the first group is the group that only gives (the richest).
(4) When balancing is initiated, the period k begins to increase and each new row in the balancing table represents a new and changed value of group wealth, due to giving and receiving, for the same number of people who owned the initial percentages.
(5) Balancing should stop when average wealth is reached. For example, if we use an example "1% ($N_1 = 1$) people own 50% of the wealth", when do we end the balancing? From (35) we have $W_1(k_B) = (W_T/N_T)N_1 = (100/100)1 = 1$ hence we look for $W_1(k_B)$ which is either 1, for $k = k_B$ (very unlikely) or two consecutive entries, one of which is slightly greater than 1, for kmin, and the other slightly smaller than 1 for $k = k_min + 1$, per (63). Plus $W1 = 1$ indicates the percentage of the total wealth after the balancing, ie. "1% people now own 1% of the total wealth". The period kB indicates the number of giving cycles (years) to reach the wealth balance. If $N1 = 5$, the algorithm stops at $W_1(k_B) = 5$, and similarly for other $N1 = 5$.
(6) We can even start from some middle point in the balancing table and check how many cycles it would take to reach a certain average wealth. In other words, when specifying an individual N_i, we can interpret ANY entry in the table as $W_i(k_B) = W_A N_i$=certain percentage of $W_T - a$. Therefore, normalized wealth tables are universal and can be applied to any community wealth normalized to 100, with a set of L groups, with arbitrary N_i.
(7) Ideally, we would know all N_i. If not, we can use the ones we know. Most likely the value for N1 will be accessible because it indicates some key information, such as "1% owns 50%". Given a specific community, these numbers can be determined by some statistical methods.

4.7. Numerical example

We now illustrate the "mean halved" MH methodology with an example for two indices, Z = 20 and 40, i.e. members of the community give one 20th or one 40th of their wealth to the community. We assume $W_T = 100$, $N_T = 100$, L=5 (6 groups), with $N_1 = 1$ (1% owns 50%) and that is the current situation of the world's wealth. Tables 1 and 2 show the normalized results. In both cases, the average wealth is $W_T/N_T =$

100/100 = 1 per community member, and that should hold for Ni in balance, when all Ni are defined. Balanced group W1 requires an average $W_A = W_1(k_B)/N_1 = 1$, i.e. $W_1(k_B) = W_A N_1 = 1$. Grey areas in the tables indicate k_{min} and $k_{min}+1$ per (63) when the algorithm stops. Table 1 shows that it is required between $k_{min} = 77$ and $k_{min}+1 = 78$ years. Table 2 for Z = 40 shows that it takes about two times longer for the balance compared to Z = 20, i.e. $k_{min}+1 = 155$ and $k_{min}+1 = 156$ years. Continuing with the example, we assign the remaining values N2 through N6 using world inequality data, per references at the back of he paper. The details follow:

$$N_1 = 1, N_2 = 2.5, N_3 = 4.5, N_4 = 6.5, N_5 = 6.5, N_6 = 79 \tag{4.45}$$

i	1	2	3	4	5	6	
k							W_T
1	50	25	12.5	6.25	3.125	3.125	100
2	47.5	25	13.125	6.875	3.59375	3.90625	100
3	45.125	24.9375	13.6875	7.46875	4.0546875	4.7265625	100
4	42.86875	24.81875	14.190625	8.03125	4.506640625	5.583984375	100
5	40.7253125	24.64953125	14.63742188	8.562617188	4.948554688	6.4765625	100
6	38.68904688	24.4351875	15.03085547	9.063074219	5.379486328	7.402349609	100
......	......	......	......	......	......	......	......
75	1.123354413	2.749262116	4.569656386	6.26005174	7.589933109	77.70774224	100
76	1.067186692	2.63988287	4.42394705	6.102677305	7.444751823	78.32155426	100
77	1.013827358	2.534568394	4.282086603	5.947810568	7.300214729	78.92149235	100
78	0.96313599	2.433185658	4.144019325	5.795490731	7.156434602	79.50773369	100
79	0.91497919	2.335604775	4.0096872	5.645751098	7.013517592	80.08046014	100
80	0.869230231	2.241699016	3.879030199	5.498619403	6.87156342	80.63985773	100

TABLE 1. $W(k+1)$ *za* $W_T = 100, N_T = 100, N_1 = 1, Z = 20, L = 5$

i	1	2	3	4	5	6	
k							W_T
1	50	25	12.5	6.25	3.125	3.125	100
2	48.75	25	12.8125	6.5625	3.359375	3.515625	100
3	47.53125	24.984375	13.109375	6.8671875	3.591796875	3.916015625	100
4	46.34296875	24.95390625	13.39101563	7.1640625	3.822119141	4.325927734	100
5	45.18439453	24.9093457	13.65780762	7.453132324	4.050202637	4.745117188	100
6	44.05478467	24.85141699	13.91013171	7.734411255	4.275915344	5.173340027	100
......	......	......	......	......	......	......	......
153	1.065789876	2.609818799	4.353727118	5.995139921	7.313418592	78.66210569	100
154	1.039145129	2.557895702	4.284167862	5.919324972	7.242554151	78.95691218	100
155	1.013166501	2.506937624	4.215532019	5.844128123	7.171874997	79.24836074	100
156	0.987837338	2.456928764	4.147812729	5.769553576	7.101394052	79.53647354	100
157	0.963141405	2.407853512	4.081003004	5.695605192	7.031123848	79.82127304	100
158	0.93906287	2.359696442	4.015095731	5.622286501	6.961076536	80.10278192	100

TABLE 2. $W(k+1)$ *za* $W_T = 100, N_T = 100, N_1 = 1, Z = 40, L = 5$

Balance Table 3 was created based on these data.

i	1	2	3	4	5	6	
N_i	N_1 1	N_2 2.5	N_3 4.5	N_4 6.5	N_5 6.5	N_6 79	N_T 100
k							W_T
155	1.013166501	2.506937624	4.215532019	5.844128123	7.171874997	79.24836074	100
156	0.987837338	2.456928764	4.147812729	5.769553576	7.101394052	79.53647354	100

TABLE 3. $W(k+1)$ *za* $W_T = 100, N_T = 100, Z = 40, L = 5$

As an illustration of the flexibility of the MH model, in the following example we choose L = 6 (7 groups) and disassemble W_6 from the above example into two groups,

”poor” and ”very poor”. The corresponding N numbers are:

$$N_1 = 1, N_2 = 2.5, N_3 = 4.5, N_4 = 6.5, N_5 = 6.5, N_6 = 71, N_7 = 18. \quad (4.46)$$

Figure 5.1 (Lorenz curve) shows all the details for the 7 groups according to the model of this paper. Comments follow.

(1) The MH balancing algorithm transforms the original Lorenz curve in Figure 5.1 into an ideal Lorenz line. Mathematically, the MH balancing algorithm changes a non-linear curve to a linear one using recursion in a specified number of steps. Depending on the value of Z, the balancing lasts from 6-7 years (Z=2, 50% giving), 155-156 years (Z=40, 2.5% giving) to 390-391 years (Z=100, 1% benefits).

(2) Figure 5.1 clearly identifies the corresponding middle class W_3, W_4 and W_5, between very rich (W_1), rich (W_2), and poor (W_6), and very poor (W_7), (¡ $1 per day [16]), no giving group. W_4 can be considered as a middle of the middle class. As the number of wealth groups is increased (larger L) the results are more reliable. In this paper we show results for L =6 and L= 7. Figure 5.1 gives a precise world Lorenz curve and it also shows our MH model with W_i groups clearly indicated.

(3) Figure 5.1 shows middle class in the middle of the curve. Around 17,5% uf the population is in the middle class and they own 22% of W_4. “Middle“ middle class W_4 is very close to the ideal line w = pn + 100. The slope p for W_4 can be calculated from Figure 5.1 as p = (12,5-6,25)/(7,6-13,4) = - 1,0776 which is very close to the ideal p = -1 Lorenz line. On the other hand the whole middle class (W_3, W_4 and W_5) has p = (25-3.125)/(3.5-20.4)=-1.29, relatively close to p=-1. The importance of middle class is well known in economics. Between 1971 and 2021 the USA middle class was reduced from 61% to 50%. World-wide at this moment there is around 17.1% middle class of the total population [16]. We conclude that our MH wealth model is a reliable mathematical model of the current Lorenz curve.

5. CONCLUSION

In this paper, we present a new simple mathematical model for the Lorenz curve, which represents the distribution of the wealth of a given community. Our “mean halved” (MH) robust model of wealth distribution is easily adapted to a variety of practical situations both in the field of wealth and in some other applications, such as education, health, ecology and climate modeling, wherever there is an uneven distribution of some resource. The model is defined with the idea of defining a wealth redistribution algorithm. The algorithm actually corrects the Lorenz curve to an ideal line (and improves the Gini index). The investment index is also mentioned in the paper, and the results are in a future paper. The idea of the whole project is to show that the combination of giving and investing is better than just investing. The paper also shows the analysis of the convergence of the allocation algorithms. The algorithm assumes a fixed total amount of wealth between giving periods, as well as a fixed giving percentage. The first condition can be removed by the wealth normalization process between giving cycles, and the second giving condition can also be adjusted on the variable.

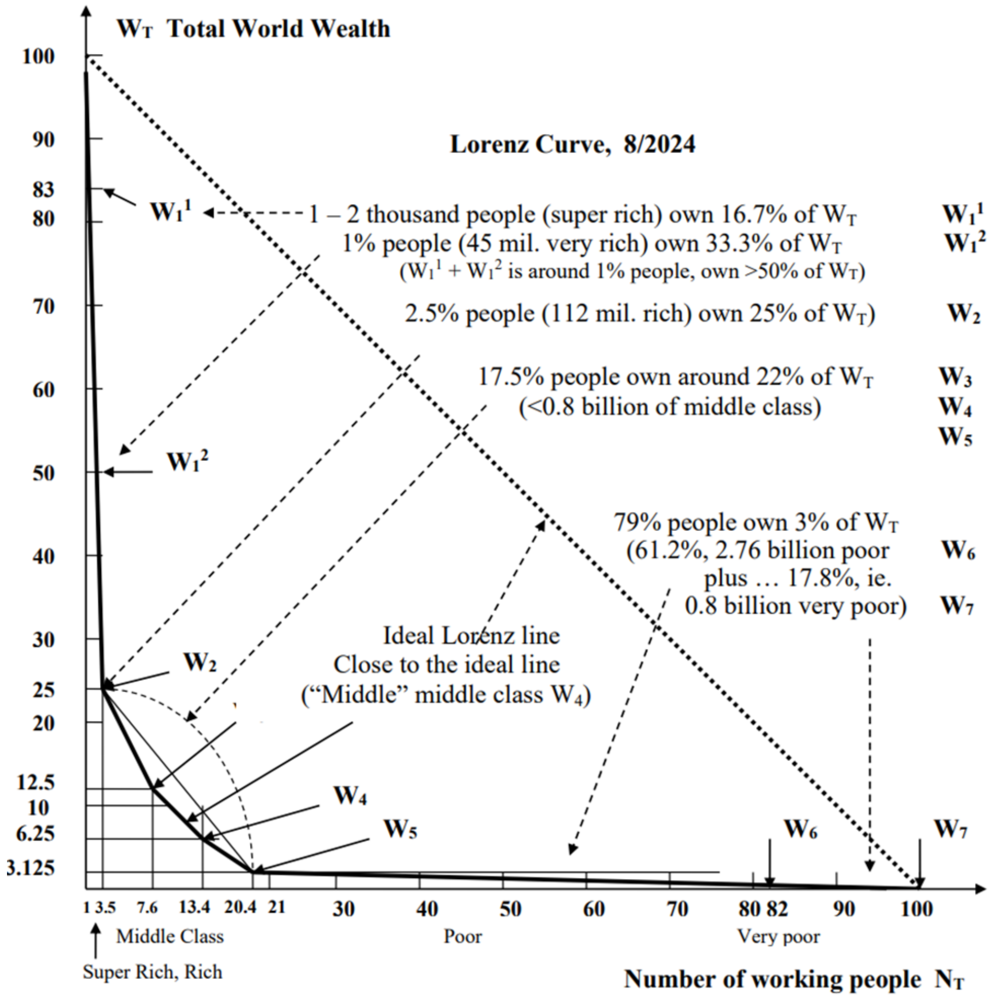

FIGURE 5.1. *World Lorenz curve per our MH model*

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