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STOCHASTIC CALCULATION OF NET LIFE INSURANCE PREMIUMS

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Dedicated to the 75th birthday of our dear Professor Mirjana Vuković

ABSTRACT. The features of life insurance are: death risk coverage (the risk of death is covered if it occurs during the contracted insurance term), long-term (life insurance contracts are concluded for several years), fixed premium (the amount of the premium is the same for the entire insurance period), savings (in many forms of this insurance, savings are also included). Life insurance contracts, among other things, differ according to the method of premium payment, namely [9], [10]:

1) insurance with premium payment at once
2) insurance with premium payment in installments - monthly, quarterly, semi-annually, annually.

Using mathematical methods based on probability and statistics, financial mathematics, stochastic models, risk theory and credibility theory, actuarial mathematics determines insurance prices, required reserves, self-retention amounts and other elements of business policy [1], [4], [5].

Therefore, regardless of the life insurance model, the principle of equivalence must be realized throughout the obligation period [7].

1. INTRODUCTION

Before moving on to stochastic approaches to calculating net premiums in life insurance, let's recall certain definitions from probability theory that we will use or rely on in our paper [8].

1.1. Random variable and discrete random variable

A random variable X is a function that assigns real numbers to the outcomes of an experiment (elements of the set Ω). The set of all values (X) that the random variable X can take is called the image of the random variable (elements are usually denoted by x_i). We are often interested in the probability p_i that the random variable X is realized by values from some set $A \subseteq \mathbb{R}$.

A discrete random variable X can have a finite $\{x_1, x_2, \dots, x_n\}$ or a countable $\{x_1, x_2, \dots, x_n, \dots\}$ set of values (X). If the probabilities $P(X = x_i)$ are known for all possible values $x_i \in \mathcal{R}(X)$, we say that the distribution of the discrete random variable X is known.

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Discrete random variables are fully determined:

- by their own picture
- and by the probability function (ie the probability $p_i = P(X = x_i), x_i \in \mathcal{R}(X)$)

1.2. Continuous random variable and distribution function

A continuous random variable is characterized by the image (X) which is not a discrete set. (a subset S of the topological space X in which every point $x \in S$ has a neighborhood in X to which no other point from S belongs is a discrete set). For example, (X) can be some interval or even the whole set of real numbers.

We define **the distribution of the continuous random variable** X by a non-negative function f, **the density function**, for which the area between the graph of this function and the x axis is equal to 1. The probability that the continuous random variable X is realized by values from some set $A \subseteq \mathbb{R}$ is equal to the area under the graph of the density function f over the set A, as shown in the following Figure 1.:

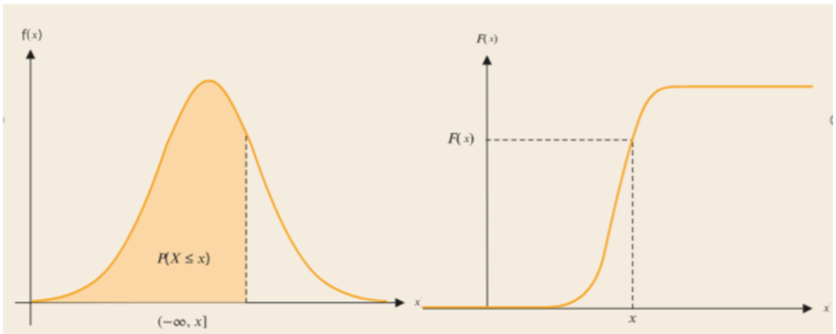

Figure 1.

Continuous random variables, which have as their image an uncountable set in the set of real numbers $\mathbb{R}$, are called **continuous random variables**.

A random variable$X : \Omega \to \mathbb{R}$, is continuous if there is a (measurable) function $f : \mathbb{R} \to \mathbb{R}$ for which:

(i) $f(x) \geq 0, x \in \mathbb{R}$,
(ii) $\int_{-\infty}^{\infty} f(x)dx = 1$,
(iii) $P(X \leq a) = \int_{-\infty}^{a} f(x)dx,\ a \epsilon \mathbb{R}$.

The function $f(x)$ is called zovemo **the density function** of X.

It follows from (iii) that for all $a,\ b \in \mathbb{R},\ a \leq b,\ P(a < X \leq b) = \int_a^b f(x)dx$.

The same applies to $P(a < X < b) = P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b)$.

The cumulative distribution function of X is the function $F : \mathbb{R} \to \mathbb{R}$, given by $F(x) = P(X \leq x)$.

The following applies:

(i) $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to \infty} F(x) = 1$,
(ii) $F(x)$ is nondecreasing
(iii) $F(x) = \int_{-\infty}^{x} f(t)dt,\ x \in \mathbb{R}$
(iv) $P(a < X \leq b) = F(b) - F(a)$,

(v) if $f(x)$ is piecewise continuous, then $F'(x) = f(x)$ except perhaps at points of discontinuity of $f(x)$.

1.3. **Deterministic characteristics of continuous random variables**

Let introduce the deterministic characteristics of continuous random variables, namely:

1. expectation
2. variance
3. standard deviation

For a continuous random variable X and its density function $f(x)$, we define **the expectation** of X (if the lower integral exists) with

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx$$

The following applies:

(i) $\mathbb{E}(\lambda X) = \lambda\mathbb{E}(X),\ \lambda \in \mathbb{R}$,
(ii) $\mathbb{E}(X+Y) = \mathbb{E}(X)+\mathbb{E}(Y)$.

The variance of X (if the lower integral exists) is defined by

$$Var(X) = \mathbb{E}\left[(X-\mathbb{E}(X))^2\right] = \int_{-\infty}^{\infty} (X-\mathbb{E}(X))^2 f(x)dx.$$

We can easily derive:

$$Var(X) = \mathbb{E}\left(X^2\right) - \mathbb{E}(X)^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \mathbb{E}\left(X^2\right).$$

The following applies:

(i) $Var(\lambda X) = \lambda^2 Var(X),\ \ \lambda \in \mathbb{R}$
(ii) $Var(X+\lambda) = Var(X)$.

The standard deviation of X (if X has variance) is defined by

$$\sigma(X) = \sqrt{Var(X)}.$$

Let's note the following:

If X is a continuous random variable with image (X) and density function $f(x)$ and $g\colon \mathbb{R} \to \mathbb{R}$, some function, then $g(x)$ is a random variable defined on the same probability space as X, has image $g(\mathcal{R}(X))$ and holds (if the lower integrals exist)

$$\mathbb{E}(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)dx,$$

$$Var\,(g(X)) = \int_{-\infty}^{\infty} (g(x)-\mathbb{E}\,(g(X)))^2 f(x)dx = \int_{-\infty}^{\infty} g^2(x)f(x)dx - \mathbb{E}\,(g(X))^2.$$

1.4. **Life insurance**

Life insurance is a long-term business and carries with it long-term risks, but much of modern actuarial risk management is focused on short-term modeling approaches [2]. A life insurance model in which the insurance premium is paid once is called single premium insurance, and a model in which the sum insured is paid multiple times at equal time intervals and in the same amount is called multiple premium insurance premium payments. Premiums can be paid multiple times in equal time intervals with different amounts, so it is insurance with multiple variable premium payments. Variability of premiums should be based on arithmetic or geometric progression. According to the duration of premium payments in relation to the duration of life, the premium is divided into lifetime and temporary. The premium is lifetime if the insured person pays it for the rest of their life, while the insured person pays the temporary premium only for a period specified in the contract.

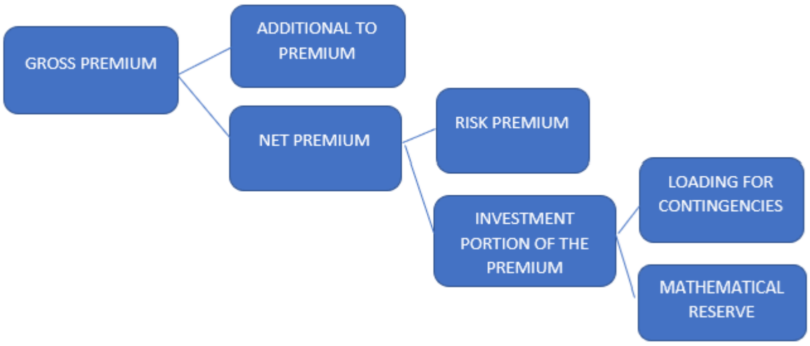

Figure 2 [11]

According to the number of payments of the insured sum, insurance is divided into capital insurance and annuity insurance. If the insured sum is paid to the insured or the beneficiary once, it is capital insurance, and if the payment occurs in several amounts and at equal time intervals, it is annuity insurance. In accordance with the differences in individual insurance models, different combinations of insurance payments and payments are created.

The success and safety of the life insurer's business depends primarily on the calculation or mathematical basis, namely the mortality tables and the interest rate. They are used to determine the net premiums, from which the funds sufficient to cover the obligations to the insured (that is, the beneficiaries) are formed.

2. STOCHASTIC APPROACH TO THE CALCULATION OF NET PREMIUM IN LIFE INSURANCE

2.1. **Preliminaries**

Mathematical laws of life insurance are based on the law of large numbers, calculus of probability and statistics, stochastic processes and credibility theory and hedging strategy. The part of mathematics used to solve and explain insurance calculation problems is actuarial mathematics. Actuarial mathematics is based on the principle, with respect for the age of the persons who enter the life insurance portfolio, the legality of stochastic processes, with the application of the time value of money.

Actuarial mathematics solves the problems of expectation of realization of the insured event using the strong law of large numbers. This law is made special by the large number of observed cases, and the greater the number of observations, the more accurate the conclusion - data, and the smaller the deviations. If an event is observed individually, it is a case, and in a large number of observations, it is a law. There are several theorems about this law, but each asserts that empirical average values converge to the expected value. These theorems are often called laws of averages.

Insurance companies are paying more attention to new phenomena that are happening and are also reflected in insurance (increased mortality of (old) persons). Variability in mortality rates within different demographic groups and/or populations within plans results in the need to review assumptions and better adapt them to specific groups. This increases the interest in new theories such as, for example, the classical theory of credibility, but also newer theories of c-credibility, such as, for example, means for adapting standard mortality tables to plans, or the portfolio included in those plans.

In addition to forecasting the occurrence of an insured event, it is also important to know the probability of the occurrence of certain insured events, so in this segment, actuarial mathematics relies on probability calculations, which are used to create mortality tables and commutative numbers (which is not the subject of this paper). Determining the probability of an adverse event in life insurance is the basis for determining the insurance premium, which follows below. Insurers with a high claims ratio usually charge high premiums. Other "competitors" set a competitive premium or accept a fixed premium to stay "in the game", otherwise they will operate below the "optimal point". Such dynamic systems, which develop in time, can also be described by non-linear Lotka-Volterra differential equations, given that it is a matter of the interaction of two types. [12] can also be applied to insurance. Regarding the type of data that affects the price of the service (insurance), the COVID-19 pandemic had an important impact on the change in the financial performance of insurance companies and on the global results in correlations between the period before and after the pandemic. [13] and [14] can also be applied to insurance.

2.2. **Mathematical model**

For life cycle modeling it is essential to know how long an individual will live. For this reason, insurers use life expectancy models to be able to calculate the probability of an individual's death at a certain age.

We start life from birth. The length of life is marked with X, x is the age of the individual. The variable X is a continuous random variable. Let $F(x)$ be the distribution function of the length of life, i.e. the distribution function of X, holds

$$F(x) = P(X \leq x),\ x \geq 0.$$

Let's define the inverse function of the distribution function $F(x)$, $s(x)$ – the distribution survival function,

$$s(x) = 1 - F(x) = 1 - P(X \leq x) = P(X \succ x),\ x \geq 0.$$

The random variable X (expected life expectancy) is completely determined by the life span distribution function $F(x)$ or the distribution survival function $s(x)$.

Thus, $F(x)$ represents the probability that the newborn will live less than x years, i.e. the probability that he will not live to age x, and $s(x)$ is the probability that the newborn will live more than x years, .e. the probability that he will live to x years. The survival function is the basis in actuarial science, and in statistics it plays the role of the mortality distribution function. The probability that a newborn dies between the years x and z, $(x < z)$ is

$$P(x \le X \le z) = F(z) - F(x) = s(x) - s(z),$$

of course, on the condition that the newborn lived to be x years old, that is

$$P(x < X < z)/X \succ x) = \frac{F(z) - F(x)}{1 - F(x)} = \frac{s(x) - s(z)}{s(x)} \quad (1)$$

The label $T(x)$ is introduced for the random variable remaining life expectancy of a person aged x in the manner

$$T(x) = X - x$$

and let ${}_tp_x$ survival probability $x+t$ for a person aged x and let ${}_tq_x = P\,(T(x) \le t)$, $t \ge 0$ the probability that a person aged x will die during the next t years, i.e. the distribution of the function $T(x)$ is

$${}_tp_x = 1 - {}_tq_x = P\,(T(x) \succ t)\,,\ t \ge 0$$

the probability that a person aged x will live to the age of $x+t$, i.e. the survival function for a person aged x.

We also introduce simpler labels

q_x - probability of death of a person aged x during the next year,

p_x - the probability that a person aged x will live to be $x+1$ years old,

${}_{t/u}q_x$ - the probability that a person aged x will live for the next t years and die in the next u, u, i.e. the probability of death occurring in the time interval $(x+t, x+t+u)$, i.e. it is valid,

$${}_{t/u}q_x = P(t < T(x) \le t =_{t+u} q_x - {}_tq_x = {}_tp_x -_{t+u} p_x$$

and according (1) is

$${}_tp_x = \frac{{}_{x+t}p_0}{{}_xp_0} = \frac{s(x+t)}{s(x)}$$

and

$${}_tq_x = 1 - \frac{s(x+t)}{s(x)}.$$

Let's also find the connection between conditional and unconditional probability

$${}_{t/u}q_x = \frac{s(x+t) - s(x+t+u)}{s(x)} = \frac{s(x+t)}{s(x)} \cdot \frac{s(x+t) - s(x+t+u)}{s(x+t)} = {}_tp_x \cdot {}_uq_{x+t}.$$

Also

$$P\,(T(x) = k) = P\,(T(x) = k+1) = 0, \ \text{ for } \ k = 0, 1, 2, 3, \ldots$$

because $T(x)$ is a continuous random variable.

Formula (1) is an equation for the conditional probability of the death of a newborn between the years x and z, with the condition that he lives to the age of x. When $x \succ z$

the probability in equation (1) still retains the property of continuity, so we can observe it as a function of x. It then describes the distribution the probability of mortality in the near future for a person who lives to age x (between time 0 and z). Analogously, the function for immediate death is obtained using the probability frequency of mortality for the case of living for the year x. Applying equation (1) fo $z = x + \triangle x$ we get

$$P\left(x < X \leq x + \triangle \frac{x}{X} \succ x\right) = \frac{F(x+\triangle x) - F(x)}{1 - F(x)} = \frac{\frac{F(x+\triangle x)-F(x)}{\triangle x} \cdot \triangle x}{1 - F(x)}$$
$$\cong \frac{F^{'}(x) \cdot \triangle x}{1 - F(x)} = \frac{f(x) \cdot \triangle x}{1 - F(x)}$$

where $f(x) = F'(x)$ is the distribution density of a continuous random variable, and the function $\frac{f(x)}{1-F(x)}$ represents the conditional probability density. For each year x it gives the value of the conditional density of the distribution of the random variable X in case of survival the same year. The function $f(x)$ is called the mortality intensity and represents the mortality rate. If we introduce the notation μ_x we have

$$\mu_x = \frac{f(x)}{1 - F(x)} = -\frac{s^{'}(x)}{s(x)} \geq 0.$$

Next, we have (with replacement)

$$\mu_y = -\frac{s^{'}(y)}{s(y)} \Longrightarrow -\mu_y dy = d(\ln s(y)) \Longrightarrow -\int_x^{x+t} \mu_y dy = \ln\left(\frac{s(x+t)}{s(x)}\right) = \ln_t p_x \Longrightarrow$$
$${}_t p_x = \exp\left(-\int_x^{x+t} \mu_y dy\right).$$

If we assume that $y = x + s$ we get:

$${}_t p_x = \exp\left(-\int_0^t \mu_{x+s} ds\right) \text{ i.e. } {}_t p_x = e^{-\int_0^t \mu_{x+s} ds}.$$

If the surviving years of life are compared with the value zero and the survival time with x we obtain:

$${}_n p_x = s(x) = \exp\left(-\int_0^n \mu_s ds\right),$$
$$F(x) = 1 - s(x) = 1 - \exp\left(-\int_0^x \mu_s ds\right)$$
$$F^{'(x)} = f(x) = \exp\left(-\int_0^x \mu_s ds\right) \cdot \mu_x = {}_x p_0 \cdot \mu_x$$

The following marks are introduced:

$\Phi(t)$ - distribution function of a continuous random variable remaining life time of a person x years of age,
T(x) = x - X, respecting $\Phi(t) = {}_t q_x$ and
$\varphi(t)$ - density of distribution of continuous random variable $T(x)$.

$$\varphi(x) = \frac{d}{dt}{}_tq_x = \frac{d}{dt}\left(1 - \frac{s(x+t)}{s(x)}\right) = \frac{s(x+t)}{s(x)} \cdot \left(-\frac{s'(x+t)}{s(x)}\right) = {}_tp_x \cdot \mu_{x+t}, \text{ for } t \geq 0$$

or

$$\varphi(x) = \frac{d}{dt}(1 - {}_tp_x) = \frac{d}{dt}{}_tp_x = {}_tp_x \cdot \mu_{x+t} \text{ otherwise.}$$

The product ${}_tp_x \cdot \mu_{x+t}$ represents the probability of mortality between the years x and $x+t$, for a person aged x years, i.e.

$$\int_0^\infty {}_tp_x \cdot \mu_{x+t}dt = 1,\ t \geq 0 \text{ holds.}$$

For a continuous random variable, the expected value is equal to a definite integral [3]:

$$\mathbb{E}[f(t)] = \int_0^\infty f(t)g(t)dt.$$

In the stochastic model, it is assumed that the interest rate is constant (the interest rate is a relative measure that describes interest, that is, the difference between the final sum of money at the end of the compounding period and the nominal value of the principal). Enter the indicator b_t in the following way:

$b_t = 1$ – if the insured risk occurs while the contract is in force

$b_t = 0$ – if the insured risk does not occur while the contract is in force

Let v_t be the discounted sum insured (the present value of the amount at which the insurance contract was concluded, i.e. the present value of the amount that will be paid to the insurance beneficiary when the insured event occurs), for the discount factor v o which the time t (from the beginning of the insurance) is related , until the liability of the insurer). Additional clarification: determining the present value with a known final (future) value is often called discounting.

It is valid $v_t = v^t$. The variables b_t and v_t are dependent on time and directly determine the random variable remaining life time of a person x years - $T(x)$. Let the nominal value of the sum insured be the random variable Z, $Z = z(t) = b_t \cdot v_t$. The expected value of the discount value of the sum insured is $E(Z)$ one-time premium in life insurance.

2.3. **Present value of one-time net premium payments**

If it is a temporary capital insurance in the event of death, the insurer's obligation is to pay the insured sum to the insured in the event of the death of the insured within the term defined in the contract. That is, if death occurs before the expiration of the term, the insurer has the obligation to pay the insured amount, otherwise it does not. Let n be the symbol for the duration of the insurance, so the nominal value of the insured sum is equal to:

$$Z = \begin{cases} v_t,\ T \leq n \\ 0, T \succ n \end{cases} \quad \text{when} \quad b_t = \begin{cases} 1, t \leq n \\ 0, t \succ n \end{cases} \quad \text{and} \quad v_t = v^t.$$

The notation $\ddot{A}_{x\overline{n}|}$ [6] is introduced for the one-time net premium of n annual capital insurance for the death of a person aged x It is equal to the expected nominal value of the sum insured.

The function $Z = z(t)$ is the density function of the random variable $T(x)$ so:

$$\ddot{A}_{x\overline{n}|} = \mathbb{E}(Z) = \mathbb{E}(z_t) = \int_0^{\infty} z_t \cdot g(t)dt = \int_0^{n} v^t {}_t p_x \mu_{x+1} dt.$$

The distribution of the random variable Z at j moment can be determined from:

$$\mathbb{E}\left(Z^j\right) = \mathbb{E}(z_T) = \int_0^{n} (v_t)^j {}_t p_x \mu_{x+1} dt = \int_0^{n} e^{-(\delta \cdot j)t} {}_t p_x \mu_{x+1} dt.$$

It follows from this equation that the j moment of the distribution of Z is equal to the one-time premium of n annual insurance in the event of death for an interest rate that is j times higher than δ, that is, for the decursive factor in the continuous increase $e^{-\delta \bullet j}$. he statement is also valid for interest at the effective interest rate.

The variance, as a measure of the dispersion of the expected value, is:

$$Var(Z) = \mathbb{E}\left(Z^2\right) - (\mathbb{E}(Z))^2 =^2 \ddot{A}_{x\overline{n}|} - \left(\ddot{A}_{x\overline{n}|}\right)^2$$

where ${}^2\ddot{A}_{x\overline{n}|}$ is a one-time net premium for an n – year period with an interest rate of 2δ.

In the case of lifetime capital insurance, the insurer must pay the sum insured to the beneficiaries upon the occurrence of the insured event. The assumptions of this stochastic model are:

$b_t = 1$ for $t \geq 0$
$v_t = v^t$ for $t \geq 0$
$Z = v^t$ for $T \geq 0.$

The symbol for the one-time net premium for life insurance $\ddot{A}_x$ is introduced and is determined:

$$\ddot{A}_x = \mathbb{E}(Z) = \mathbb{E}(z_t) = \int_0^{\infty} z_t \cdot g(t)dt = \int_0^{n} v^t {}_t p_x \mu_{x+1} dt$$

Life insurance lasts until the end of the insured person's life, so the number of years of insurance is considered infinitely large, i.e. $n \to \infty$. We know from experience that there are few people who live more than 100 years, but in general the marginal value of life expectancy, and thus of insurance, is an infinite value. This assumption does not significantly change the value of the one-time premium because:

$$\int_{100}^{\infty} v^t {}_t p_x \mu_{x+1} dt \to 0$$

If we observe the intensity of mortality μ_x as a constant μ with a certain constant interest rate $\delta = \frac{p}{100}$, the one-time lifetime insurance premium can be expressed using the following equation:

$$\ddot{A}_x = \mathbb{E}(Z) = \mathbb{E}(z_t) = \int_0^{\infty} e^{-\delta t} e^{-\mu t} \mu dt = \frac{\mu}{\mu + \delta}.$$

In the case of life insurance, the sum insured is paid to the beneficiary if the insured lives to the age for which the contract was concluded. The following applies:

$$Z = \begin{cases} 0, \; T \leq n \\ v^n, T \succ n \end{cases} \quad \text{when} \quad b_t = \begin{cases} 1, t \leq n \\ 0, t \succ n \end{cases} \quad \text{and} \quad v_t = v^t, \; t \geq 0$$

The one-time net premium for the case of survival is denoted by $A_{x\overline{n}|}$ and is equal to:

$$A_{x\overline{n}|} = \mathbb{E}(Z) = v^n{}_n p_x$$

with variance

$$Var(Z) =^2 A_{x\overline{n}|} - \left(A_{x\overline{n}|}\right)^2 = v^{2n} \cdot {}_n p_x \cdot {}_n q_x.$$

2.4. **Present value of multiple payments**

If the premiums are paid continuously until the occurrence of the insured event, and if the symbol a_x is introduced for the expected present value of all payments, we have

$$a_x = \int_0^\infty e^{-\delta t} dt.$$

Let's assume for simplicity that the payments are unitary and that they are paid over n years, we have

$$a_x = \int_0^n e^{-\delta t} dt.$$

If we denote by A_x the expectation of the stochastic discounted present value of the sum insured (in the amount of one monetary unit), we have

$$A_x = \int_0^\infty e^{-\delta t} f(t) dt,$$

where $f(t)$ is the probability density function for the remaining lifetime of the random variable T_x.

From the last two equalities we have

$$A_x = \int_0^\infty e^{-\delta t} f(t) dt = 1 - \delta a_x.$$

When the insurance is paid in multiple equal payments, the net periodic premium is calculated by the ratio,

$$NPP = \frac{A_x}{a_x} = \frac{1}{a_x} - \delta.$$

With this approach, the discount values of payments (payments) that are a function of time are equated with expected values that depend on time and the interest rate.

3. CONCLUSION

The stochastic model allows the determination of variance for selected functions related to mortality, and variance is by definition a deviation from the expected value and certainly one of the measures of risk. This means the possibility of determining the error for the calculated single premium. A positive characteristic of the stochastic model is certainly risk minimization, but it is not acceptable for practical application.

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